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Technical Report No. 1-19

SUMMARY REPORT

RESEARCH AND DEVELOPMENT STUDY

ON STRESS-STRAIN CHARACTERISTICS

OF SHELLS AND HIGH EXPLOSIVES

by

R. C. Geldmacher, A. C. Eringen

July, 1961

AD 269037

INTRODUCTION

In this report a summary is given of the work done during the second year of a Research and Development Study on Stress-Strain Characteristics of Shells and High Explosives.

In the summary of the first year's work, it was pointed out that the ordnance design engineer, in common with all engineers, is often required to develop systems for which analytic design procedures are not available and he must consequently call upon his resources in the art of design to carry through his assignments. As analytic procedures based on general physical laws are developed for ordnance problems, the engineer is able to move from the art of design to rational design and consequently to perform his work more efficiently and precisely.

The basic purpose of this study has been to investigate and develop rational procedures for the design of artillery shells. During the two years that this study has been underway, the following reports have been prepared:

TR 1-1 Experimental Procedures for Determining Shear and Dilatational Viscosities of Shell Filler Materials. By J. W. Dunkin and R. C. Geldmacher, January 1959.

TR 1-2 Axisymmetrical Buckling of Conical Shells Under Inertia Loading. By A. C. Eringen, April 1959.

TR 1-3 Stress Analysis of a Thin Cylindrical Shell with Flat Base. By A. C. Eringen, April 1959.

TR 1-4   Longitudinal and Torsional Vibration of Internally Damped Cylindrical Bars with Attached Masses. By J. W. Dunkin, May 1959.

TR 1-5   A Procedure for Computation of Stresses in Flat Base Shells. By N. F. Jordan, J. C. Samuels, J. W. Dunkin, and A. C. Eringen, July 1959.

TR 1-6   On the Axisymmetric Displacements and Stresses of a Long, Thick Cylindrical Shell Under Axisymmetric Loads. By A. C. Eringen, M. M. Stanisic, and R. J. Jezik, July 1959.

TR 1-7   Stress Analysis of a Thin Cylindrical Shell with Flat Base and Springy Filler. By L. V. Kline, August 1959.

TR 1-8   Vibrations of an Accelerating Visco-Elastic Rod. By A. C. Eringen and J. W. Dunkin, July 1959.

TR 1-9   Stress Analysis of Flat Base Shell Filled with a Springy Velocity Damped Filler. By J. C. Samuels, February 1960.

TN 1-3   Approximations for Scaling a Flat Base Shell from Titanium to Steel. By J. W. Dunkin, A. C. Eringen, and R. C. Geldmacher, February 1960.

TR 1-10 Thermal Stresses in Concentric Sperical Layers. By M. M. Stanisic, A. C. Eringen, and R. J. Jezik, February 1960.

TR 1-11 Long Cylindrical Shell with Visco-Elastic Filler. By A. C. Eringen, February 1960.

TR 1-12 Experimental Investigation of Visco-Elastic Properties of Filler Materials. By J. W. Dunkin and T. M. Roach Jr., February 1960.

TR 1-13 Measurements of Gun and Shell Strains During Firing. By R. C. Geldmacher and T. M. Roach Jr., February 1960.

TR 1-14   Final Report (Phase 1) Research and Development Study on
Stress-Strain Characteristics of Shells and High Explosives.
By R. C. Geldmacher and A. C. Eringen, February 1960.

TN 1-4   A Simplified Stress Analysis of Two-Piece Shell with Flat
Base. By N. F. Jordan, A. Jones, and J. W. Dunkin, August
1960.

TR 1-15   Calculations on the Axisymmetric Displacements and Strains
of a Long, Thick Cylindrical Shell Under Axisymmetric Loads.
By R. J. Jezik and N. F. Jordan, May 1961.

TR 1-16   An Elastic-Plastic Analysis of a Thin-Walled Circular
Cylinder under Axially Symmetric Loading. By A. Jones,
A. C. Eringen, and M. M. Stanisic, May 1961.

TR 1-17   Procedure for Development of a Digital Computer Program
for Flat Base Shell Analysis. By R. C. Geldmacher and
T. M. Roach Jr., June 1961.

TR 1-18   The Deformation of Filled Shells During Firing. By R. C.
Geldmacher and T. M. Roach Jr., July 1961.

TR 1-19   Summary Report, Research and Development Study on Stress-
Strain Characteristics of Shells and High Explosives. By
R. C. Geldmacher and A. C. Eringen, July 1961.

The 21 reports listed above along with previously reported
investigations [1], [2], [3], [4], [5][1] represent an integrated
approach to a study of stress-strain characteristics of shells and
high explosives. The over-all problem is one of considerable
complexity and substantial progress has been made and important new
analytical tools are now available to the ordnance engineer.

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[1] Numbers in square brackets designate references listed at the end
of this report.

A complete experimental and theoretical elastic analysis has been made of the flat base shell and a procedure for programming the analysis on a digital computer has been provided. This work is contained in TR 1-3, TR 1-5, TN 1-3, TN 1-4, TR 1-13, TR 1-17, and TR 1-18. The engineer, therefore, now has techniques for thorough analysis of his designs and with the digital computer procedure, a means of analyzing his designs in a matter of minutes. With these new tools he should be able to develop end items which will be less costly to produce and which will perform their intended functions more efficiently.

Excellent progress has also been made under the present contract on three principal remaining problems, namely: the behavior of filler materials, the interaction between filler and shell, and yielding of the shell in the vicinity of the rotating band.

In the present report a summary is given of work done under the current contract extension. This includes procedures for using the digital computer in flat base shell analysis, the study of yielding of shells in the vicinity of the rotating band, study of the interaction between filler and shell, preparation of design curves for thick wall tubes with ring loading, a simplified stress analysis of a two-piece shell with flat base, and consideration of the desirability of securing a triaxial test machine for determining filler properties.

DIGITAL COMPUTER PROGRAM FOR ANALYSIS
OF EMPTY FLAT BASE SHELLS

The analysis of flat base shells presented in TR 1-3 (April 1959) has been developed in TR 1-17 (June 1961) into a step by step procedure for digital computer calculation of elastic displacements and stresses in the shell walls and base.

The engineer need only present geometry, magnitude of loading, and material properties to the digital computing center in order to obtain a complete analysis of his shell design.

At the present time only the analysis of an empty shell has been provided because experimental work concerning the effect of fillers is still in progress. However, all steps in the computation which are common for both filled and unfilled shells have been put down in anticipation of the time when the experimental work will be completed.

The geometry of the shell and distribution of loading is shown on page 6. In this figure, P_g is gas pressure, P_b is band pressure, and kw is the filler restoring force.

The analysis of the problem requires the simultaneous solution of the three differential equations for the cylindrical bays and the differential equation for the base. In the process of solving these equations, it becomes necessary to solve a set of 14 simultaneous algebraic equations of which three are non-linear.

The solution of the 14 equations is obtained by means of a procedure which begins with the inversion of two 11 by 11
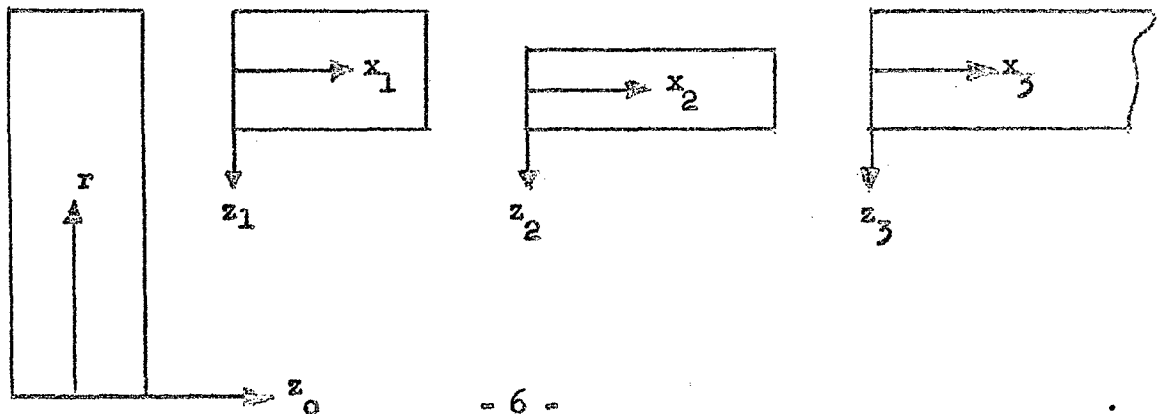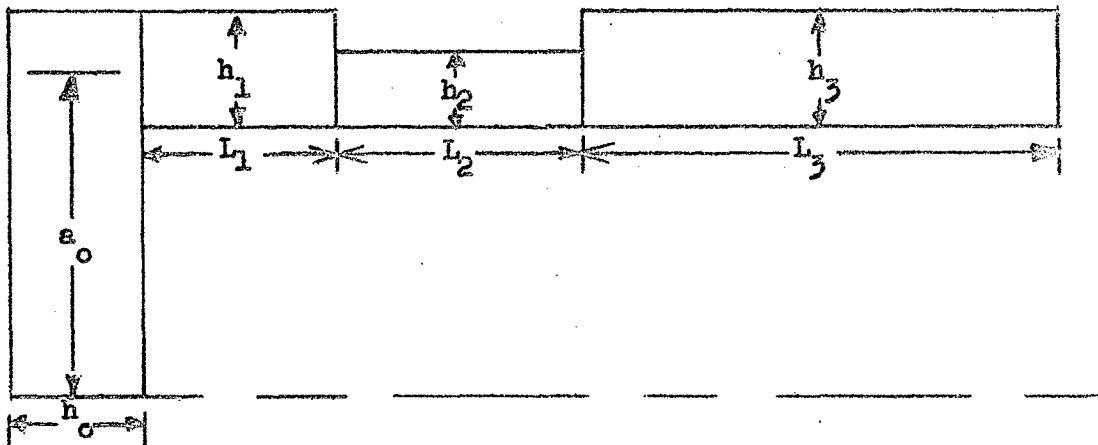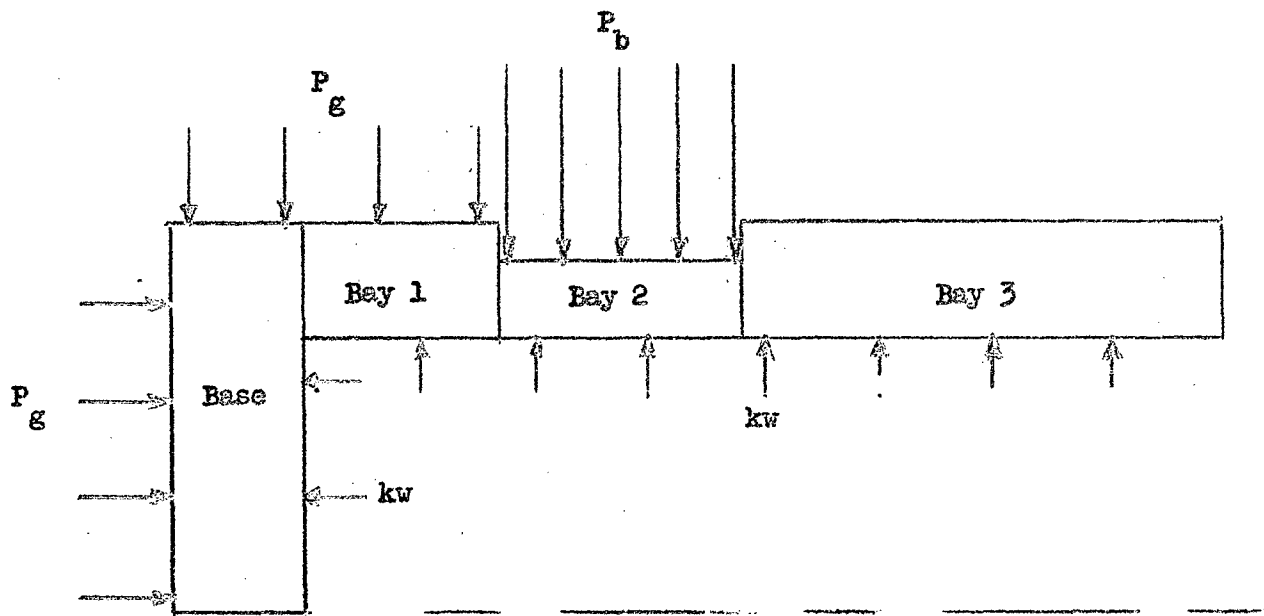
matrices from which an approximate value for the 14 required quantities may be obtained. Once the approximate values have been obtained, a cylical process is used to refine the approximations.

YIELDING OF SHELL IN VICINITY OF ROTATING BAND

There is substantial evidence that standard shell designs yield in the vicinity of the rotating band during firing [4], [5]. There is further evidence that such local yielding is not necessarily detrimental to the performance of filled shells since fillers do not ordinarily constrain the deflection of the shell walls even when large deflections occur (see TR 1-13 and TR 1-18).

Thus, an analysis of yielding in the vicinity of the rotating band is of considerable importance to the ordnance engineer since it would permit him to develop rational designs that could incorporate local yielding in a way that could lead to a reduction in the amount of material used in the manufacture of shells.

In TR 1-16 is presented an elastic-plastic analysis of a thin-walled circular cylinder under axially symmetric loading. The analysis assumes an incompressible media, a condition of plane stress, a stress distribution as shown on page 8, and a Tresca yield criteria.

A non-linear differential equation is obtained and is simplified by means of a perturbation technique which assumes the yielding to be localized and loading to be near the yield load.

Assumed Stress Distribution
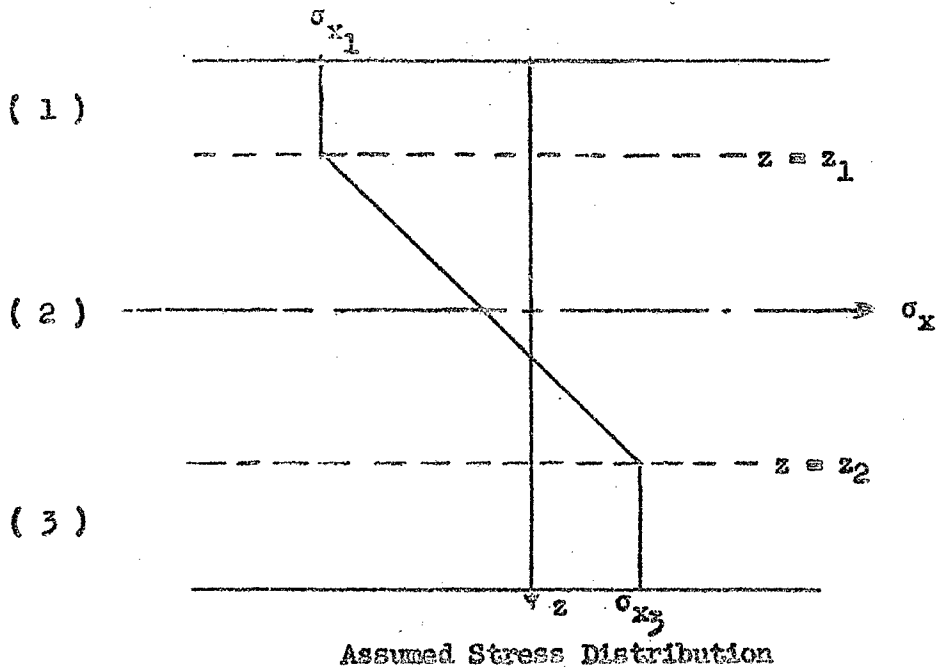
Some results from this theory are shown on pages 9 and 10. In these curves the zero abscissa corresponds to the position of the load.

The curves on page 9 show the elastic-plastic interface for various loadings. In the regions above and below the pairs of curves having the same parameter ϵ, yielding occurs; the region between corresponding curves is elastic. The parameter ϵ represents a small increase above the load P_0 at which yielding begins. Thus, $P = P_0(1 + \epsilon)$ where P is the actual line load applied to the mid-surface of the cylinder and measured in pounds per inch of circumference.

- 8 -

Elastic - Plastic Interface
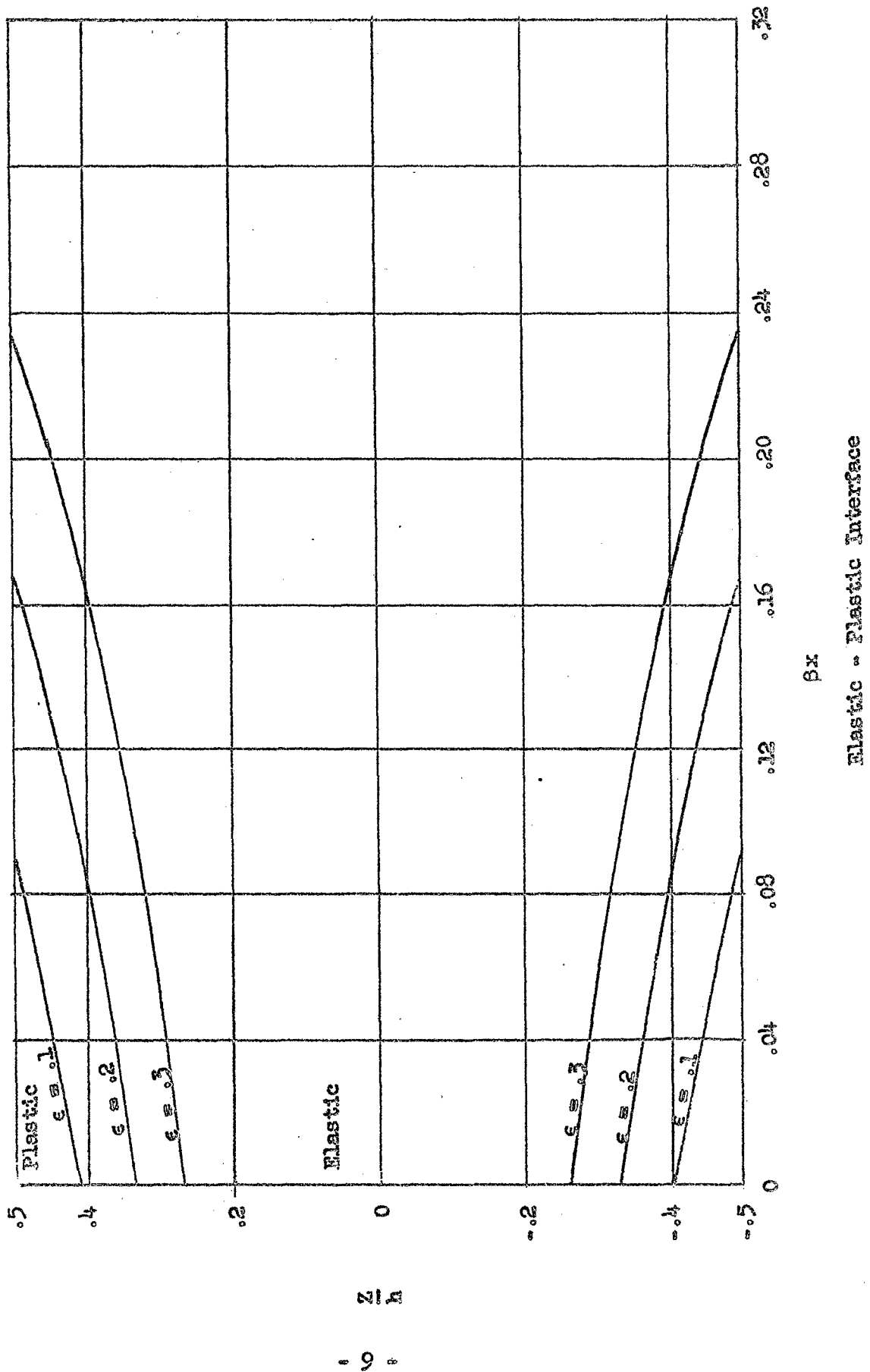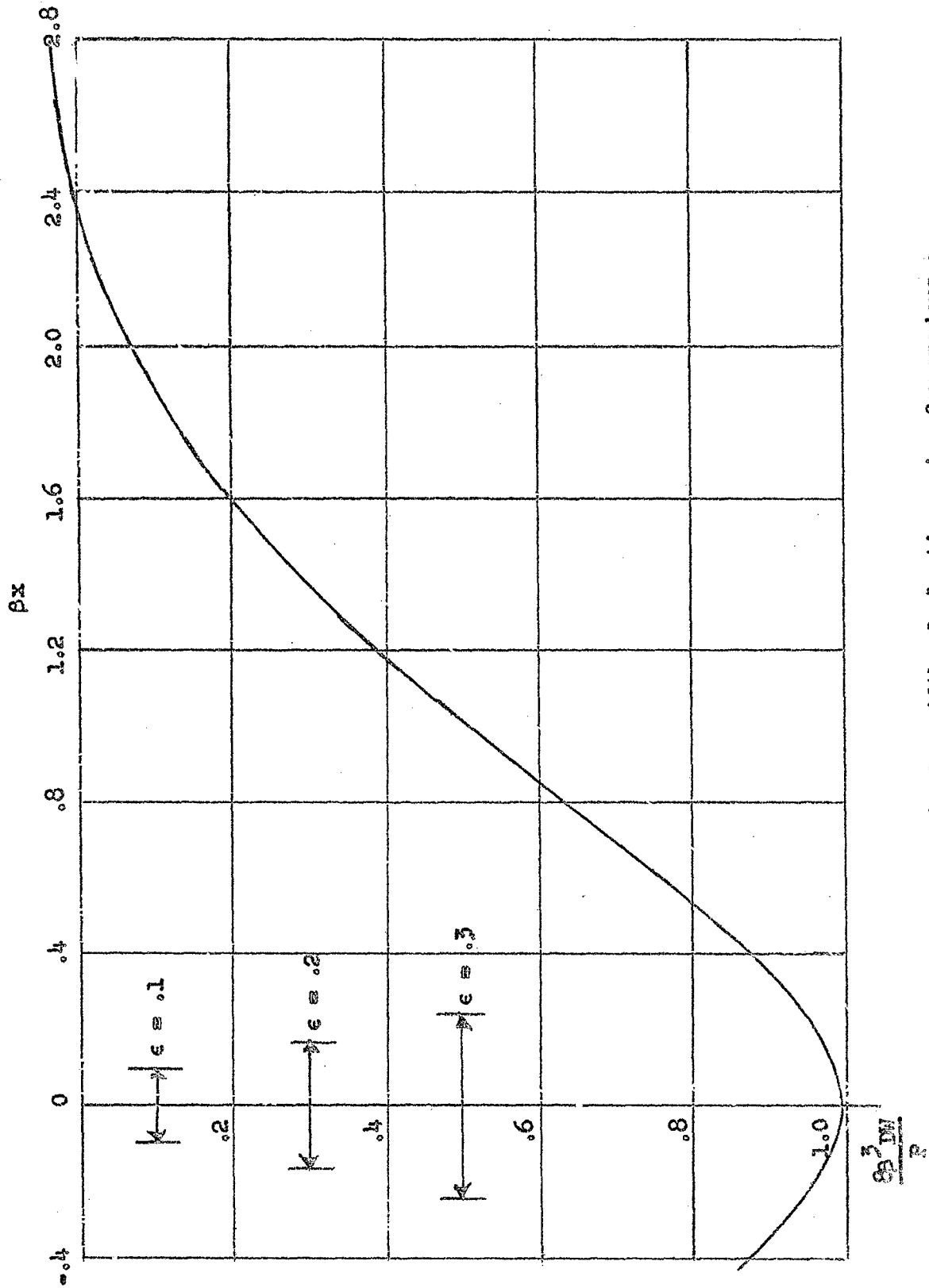
Deflection curve showing width of elastic region for various ε

The value of ϵ for a particular load P may be calculated from

$$\epsilon = \frac{3P}{2h^2 \beta \sigma_o} - 1$$

where P is defined above and

h	thickness of shell
β	$[\dfrac{9}{4R^2 h^2}]^{1/4}$
R	radius of shell (to neutral axis)
σ_o	yield stress in simple tension

The ordinates of the curves are in terms of z/h where z is the distance from the neutral axis of the shell to the point in question.

The deflection curve of the steel and the width of the plastic region for various values of ϵ is shown on page 10. The ordinates are in terms of the quantity $\dfrac{8\beta^3 Dw}{P}$ where β and P have been defined and

w	shell deflection, positive inward
D	$\dfrac{Eh^3}{9}$
E	Youngs modulus

INTERACTION BETWEEN SHELL AND FILLER

Reports TR 1-7, TR 1-9, TR 1-11, TR 1-13, and TR 1-18 are concerned with the interaction between shell and filler. These reports present theoretical and experimental work concerned with various approaches to the analysis of shell-filler systems.

The work done during the present contract extension has been experimental and is reported in TR 1-18. Earlier, measurements during firing were made of shell strains in the vicinity of the rotating band for a series of filled and unfilled shells; this work was reported in TR 1-13. The filler used in this earlier study was a PBX simulant. The maximum shell deflections as calculated from the measured strains were on the order of .008 inches for both filled and unfilled shells and it was concluded that the filler gave no support to the shell walls.

The experiments summarized in the present report were performed as a continuation of the earlier work briefly described above. Specifically, the purposes of the present work were: 1) to develop techniques for measuring shell strains under conditions of full acceleration, 2) to initiate a study of the behavior of shell fillers under conditions of full acceleration, 3) to compare the shell wall deflections of filled and unfilled shells under conditions of greater deflection than in the previous series of measurements.

The materials used in the experiments are listed below:

1) Shell: M101 machined to the flat base form shown on page 14. The initial dimensions of the shell are shown on page 15.

A photograph of the modified shell is shown on page 16. The rotating band was not modified.

2) **Filler:** Inert filler D.

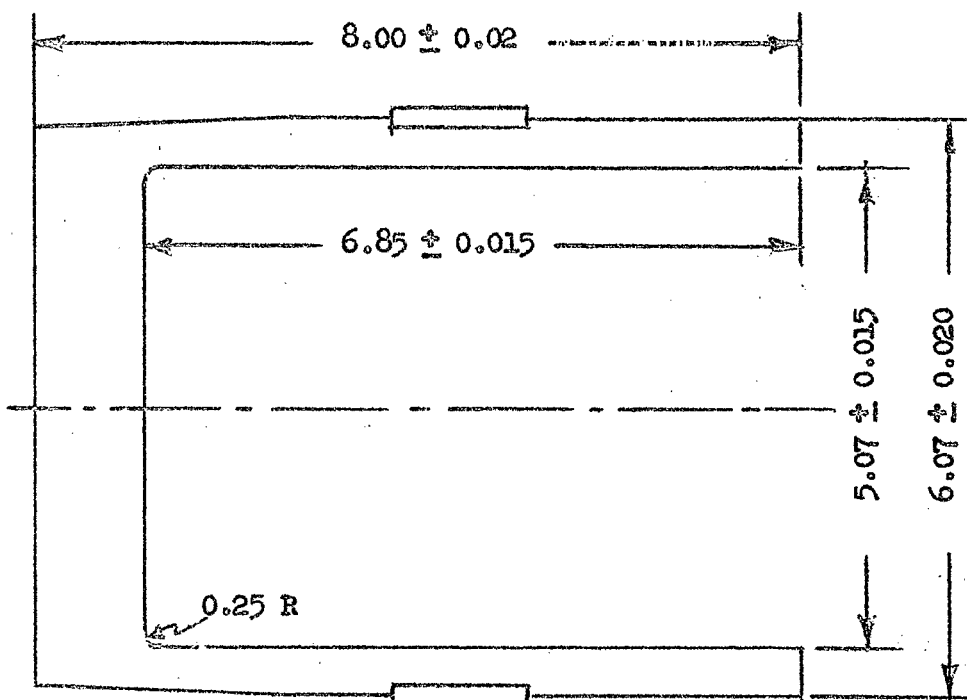3) **Gun:** Modified 155 mm M1 howitzer described in TR 1-13, a photograph is shown on page 17.

4) **Instruments:** Shell strain measurements were made using SR-4 C-7 electric strain gages. Chamber gas pressure was measured with an HF-1 pressure gage installed in the breech block. Balanced bridge circuits were used with both the strain gages and the pressure gage. Bridge output was received by an Electronic Tube Corporation four channel cathode ray oscillograph equipped with high gain D.C. amplifiers. The cathode ray traces were photographed by means of a General Radio 35 mm strip camera.

A series of eight shells was fired. Two circumferential strain measurements were made on the interior of each shell directly under the mid-point of the rotating band.

Four shells were fired at high and four at low velocity and acceleration. In each group of four, two shells were empty and two were filled. By providing special support for the gage lead wires inside the shell, it was possible to obtain measurements at accelerations corresponding to muzzle velocities of 1700 feet per second.
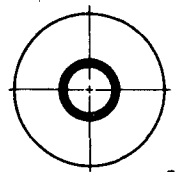
A table of maximum strains, maximum deflections, corresponding gas pressures, and corresponding velocities are shown on page 18.

8.00 ± 0.02

6.85 ± 0.015

5.07 ± 0.015

6.07 ± 0.020

0.25 R

MODIFIED M 101 SHELL

(Exterior Dimensions Unchanged)

DWG. NO. 22
U.S. GAGE

COVER, BASE 75-4-80F16
STEEL SHEET, COLD-ROLLED
TEMPER NO.3 OR 4, FINISH NO.1
SCALE, INCHES ½

VIEW B
BASE COVER GROOVE ALTERNATIVE
NOTE AC
SCALE, INCHES ½

VIEW A
SCALE, INCHES ½

BAND, ROTATING
GILDING METAL
FINISH ALL OVER ½
NOTE N

MIN MECHANICAL PROPERTIES
YS (.1% SET) ----------- 45,000 PSI MIN
ELONG IN 2 IN. ---------- 18%

BODY SHELL 75-4-80A7
FORGING STEEL
FINISH ALL OVER .250 ̷
SCALE, INCHES ½
NOTE K
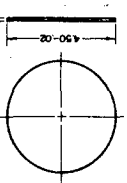
ASSEMBLY AND MARKING DIAGRAM 75-4-80E17
LESS LIFTING PLUG AND GROMMET

EMPTY

SUPERSEDES 75-4-80 WITH CHANGE JUNE 7, 1956

16

Round No.	Velocity (feet/sec.)	Maximum $\epsilon_1 \left(\frac{\mu \text{ in.}}{\text{in.}}\right)$	Maximum $\epsilon_2 \left(\frac{\mu \text{ in.}}{\text{in.}}\right)$	Average of Maximum Strains	Gas Pressure (psi)	Average Maximum Shell Deflection (in.)	Filler
5	1658	6150	6500		---		None
6	1678	6890	6650	6550	4350	16.1/1000	None
7	811	6375	6350		---		None
8	876	7050	7070	6711	2450	17/1000	None
9	715	5950	---		2000		Inert D
10	668	6750	---	6350	3800	16.1/1000	Inert D
11	1608	6425	6400		6100		Inert D
12	1599	6400	6160	6340	5220	16.1/1000	Inert D

The thermal coefficient of expansion of unit filler D was not on record at the Arsenal but is estimated to be approximately $3(10^{-4})$ inches per inch per degree centigrade when averaged over the applicable temperature range. The shrinkage of the radius of the filler as it solidified was estimated to be on the order of .05 inches.

In the table on page 18 the strains ϵ_1 and ϵ_2 are those measured on the interior of the shell. The magnitude of the strains indicates that the shell in this region was well into the yielding range. The maximum deflections of .016 inches are twice as great as in the earlier series of measurements.

Since the shell deflections are considerably less than the estimated shrinkage of the filler, it is not believed that the filler gave support to the deflected shell and this is verified by the measurements on corresponding filled and unfilled shells.

DESIGN CURVES FOR THICK WALL TUBES WITH RING LOADING

The determination of the displacement and stress field in a homogeneous, isotropic, elastic, circular cylindrical tube subject to axi-symmetric external loading requires the solution of the biharmonic equation

$$\nabla^2 \nabla^2 \Phi = 0$$

subject to boundary conditions

$$\sigma_{rr}(r,a) = \sigma(r)$$

$$\sigma_{rz}(r,a) = \tau(r) \qquad\qquad \text{on } r = a$$

$$\sigma_{rr}(r,b) = \sigma_{rz}(r,b) = 0 \qquad\qquad \text{on } r = b$$

Here

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

and $\sigma_{rr}(r,z)$ and $\sigma_{rz}(r,z)$ are the radial and longitudinal tangential stress components at a point with cylindrical coordinates (r,z) and $r = a$ and $r = b$ are the external and internal surfaces of the cylinder. In the case of radial band loading $\sigma(r)$ is prescribed and $\tau(r) \equiv 0$.

The stress components are calculated by

$$\sigma_{rr} = \frac{\partial}{\partial z}\left(\nu\nabla^2\Phi - \frac{\partial^2\Phi}{\partial r^2}\right)$$

$$\sigma_{\theta\theta} = \frac{\partial}{\partial z}\left(\nu\nabla^2\Phi - \frac{1}{r}\frac{\partial\Phi}{\partial r}\right)$$

$$\sigma_{zz} = \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2}\right]$$

$$\sigma_{rz} = \frac{\partial}{\partial r}\left[(1-\nu)\nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2}\right]$$

$$\sigma_{r\theta} = \sigma_{\theta z} = 0$$

where ν is Poisson's ratio. The solution of the problem (valid
for small deformations) is possible[1]; however, it leads to compli-
cated expressions. As a consequence, a simpler method, devised for
moderately thick cylinders, has been developed. This method is
described in TR 1-15.

It is believed that for cylinders having thickness to
average radius ratio greater than 1/5 the results of TR 1-15 would
be adequate for all engineering purposes. Moreover, the theory of
elasticity method outlined above cannot be applied to cylindrical
shells of finite lengths, while the simplified method of approach
can be used for this purpose.

Calculations made from theoretical results are presented
in TR 1-15. Three forms of loading were used: a concentrated ring
load, a uniform pressure acting on the infinite left-hand portion
of the shell, and a uniform pressure confined to the region $[-\eta_o, \eta_o]$
on the axis of the shell.

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[1]Earlier the problem was treated by this method. However, the
results obtained required heavy computer work and, therefore, the
solution obtained was abandoned in favor of the method of approach
presented in TR 1-15. Watertown Arsenal report WAL No. 893/172,
brought to our attention recently, deals with the above approach.
The solution of a similar problem for a visco-elastic cylinder using
the elasticity method is given in our TR 1-11. Computation work
carried out in the Watertown Arsenal report is for shell thicknesses
that are thicker than the range valid for the analysis in TR 1-15.
Comparison with the Watertown Arsenal report was made by extrapolating
their data to a thinner shell (see Fig. 15, TR 1-15).

For each type of loading, dimensionless ratios involving the radial displacement and dimensionless ratios involving the circumferential strain are plotted against distance along the axis. In addition to the graphs, tables are also presented which contain precise values of strain or deflections at various points.

In all calculations, the following material constants, parameters, and defined quantities were used:

Young's modulus	$E = 30 \times 10^6$
Shear modulus	$G = 12 \times 10^6$
Transverse Shear Constant	$k = 5/6$
Poisson's Ratio	$\nu = 0.3$
Shell thickness	h
Width of rotating band	$2\eta_0$
Radius to middle surface of shell	d
Longitudinal distance along shell axis measured from origin	z

$$a = \frac{1}{h} \sqrt{ \frac{h}{d} \sqrt{3(1-\nu^2)} - \frac{E}{4kG} \left(\frac{h}{d}\right)^2 }$$

$$b = \frac{1}{h} \sqrt{ \frac{h}{d} \sqrt{3(1-\nu^2)} + \frac{E}{4kG} \left(\frac{h}{d}\right)^2 }$$

Concentrated Ring Load

$$\frac{-W\,(\Omega)\,E}{2.73\ P}$$

h/d / Ω	0.08	0.1	0.125	0.15	0.175	0.2
0.0 Ω	10.225	7.2831	5.1837	3.9221	3.0960	2.5208
0.5	8.3653	5.9528	4.3000	3.1937	2.5164	2.0447
1.0	5.1186	3.6319	2.6888	1.9373	1.5208	1.2318
1.5	2.3860	1.6897	1.2738	0.8965	0.7022	0.5673
2.0	0.6768	0.4807	0.3469	0.2568	0.2018	0.1634
2.5	-0.1376	-0.0927	-0.1149	-0.1597	-0.0315	-0.0236
3.0	-0.4550	-0.3125	-0.3047	-0.1585	-0.1144	-0.0936
3.5	-0.3524	-0.2430	-0.2422	-0.1219	-0.0417	-0.0726
4.0	-0.2316	-0.1601	-0.1624	-0.0792	-0.0334	-0.0463

Uniform Pressure $(-\infty < \Omega \leq 0)$

$$\frac{-2 \Pi E W(\Omega)}{P_g^*}$$

$\frac{h}{d}$ / Ω	0.08	0.1	0.125	0.15	0.175	0.2
-4.0	12.571	10.052	8.0560	6.6991	5.7408	5.0224
-3.0	12.780	10.215	8.2351	6.8016	5.8251	5.0930
-2.0	12.770	10.231	8.2828	6.8053	5.8268	5.0931
-1.0	11.142	8.9850	7.2349	5.9838	5.1270	4.4837
0.0	6.2500	5.0000	4.0000	3.3333	2.8571	2.5000
1.0	1.3584	1.0152	0.7651	0.6829	0.5873	0.5156
2.0	-0.2699	-0.2310	-0.2829	-0.1386	-0.1126	-0.0431
3.0	-0.2800	-0.2150	-0.2351	-0.1350	-0.1108	-0.0930
4.0	-0.0705	-0.0523	-0.0559	-0.0324	-0.0265	-0.0224

Uniform Pressure $(-\eta_o \leq z \leq \eta_o)$

$h/d = 0.08,\quad \Omega' = 0.07138$

Ω	$\dfrac{-8\Pi\, E\, W(\Omega)}{P_g^{*}}$	$\dfrac{-E\,\epsilon_\theta(\Omega)}{P_g}$
0.0	3.5850	0.8618
0.5	2.9196	0.7018
1.0	1.8070	0.4344
1.5	0.8358	0.2009
2.0	0.2381	0.0572
2.5	-0.0470	-0.0113
3.0	-0.1592	-0.0383
3.5	-0.1206	-0.0290
4.0	-0.0812	-0.0195

CONCENTRATED RING LOAD
P = LBS. PER CIRCUM. INCH

$\dfrac{-W(\Omega)E}{2.73P}$

$\Omega = az$

$a = \sqrt{\dfrac{h}{d}}\sqrt{3(1-\nu^2)} - \dfrac{E}{4KG}\left(\dfrac{h}{d}\right)^2\Big/ h$

$E = 30 \times 10^6$

$G = 12 \times 10^6$

$K = 5/6$

$\nu = 0.3$

h/d = 0.08

h/d = 0.125

h/d = 0.175

Ω

$\dfrac{-2\pi E W(\Omega)}{P_g^*}$

UNIFORM PRESSURE FROM 0
TO $(-\infty)$
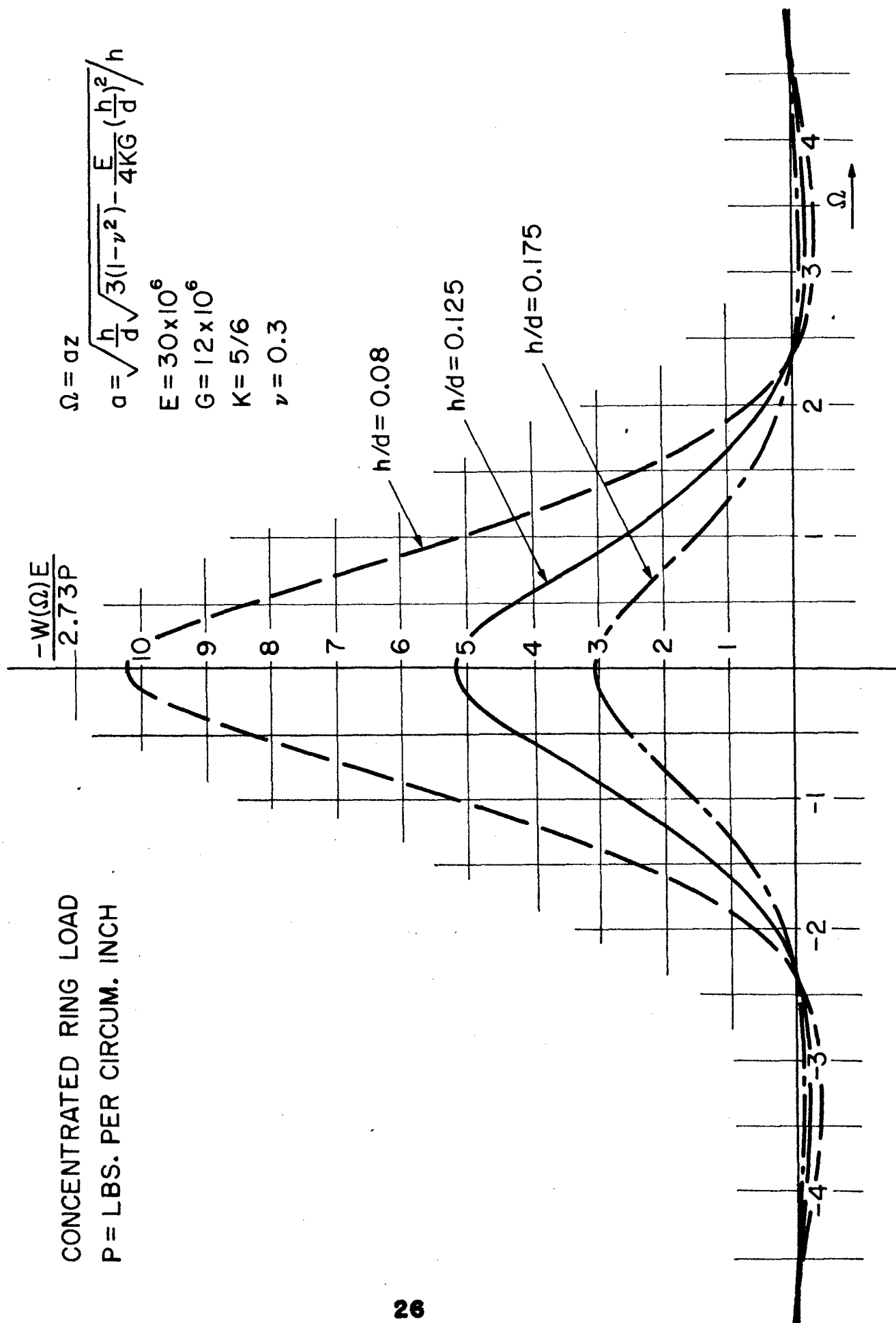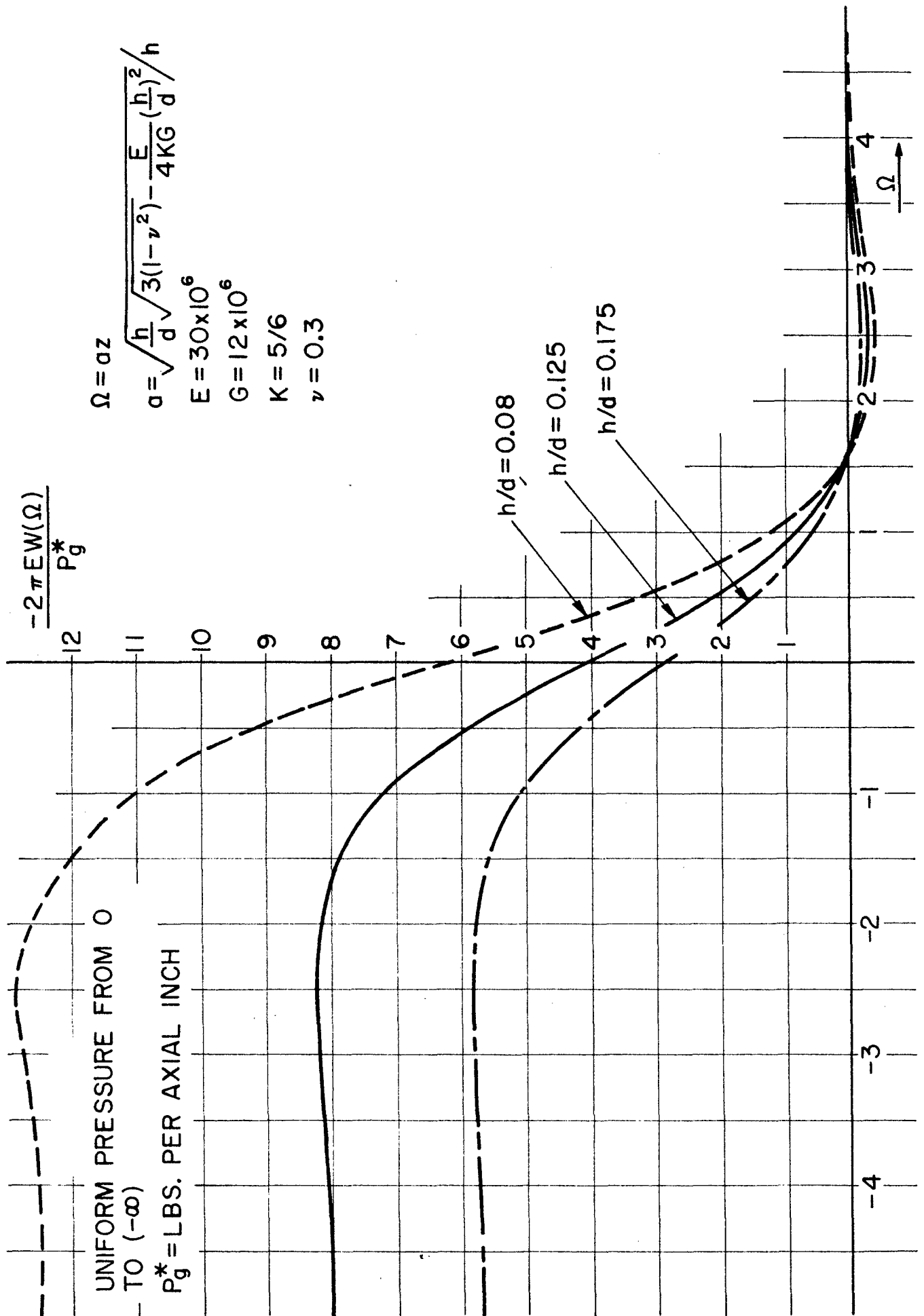P_g^* = LBS. PER AXIAL INCH

$\Omega = az$

$a = \sqrt{\dfrac{h}{d}} \sqrt[4]{3(1-\nu^2)} - \dfrac{E}{4KG}\left(\dfrac{h}{d}\right)^2 \Big/ h$

$E = 30 \times 10^6$

$G = 12 \times 10^6$

$K = 5/6$

$\nu = 0.3$

h/d = 0.08

h/d = 0.125

h/d = 0.175

Ω

27

UNIFORM PRESSURE FROM $(-\eta_0)$ TO η_0

$P_g^* =$ LBS. PER AXIAL INCH

$$\frac{-8\pi E W(\Omega)}{P_g^*}$$

$\Omega = az$

$a = \sqrt{\dfrac{h}{d}} \sqrt{3(1-\nu^2)} - \dfrac{E}{4KG}\left(\dfrac{h}{d}\right)^2 / h$
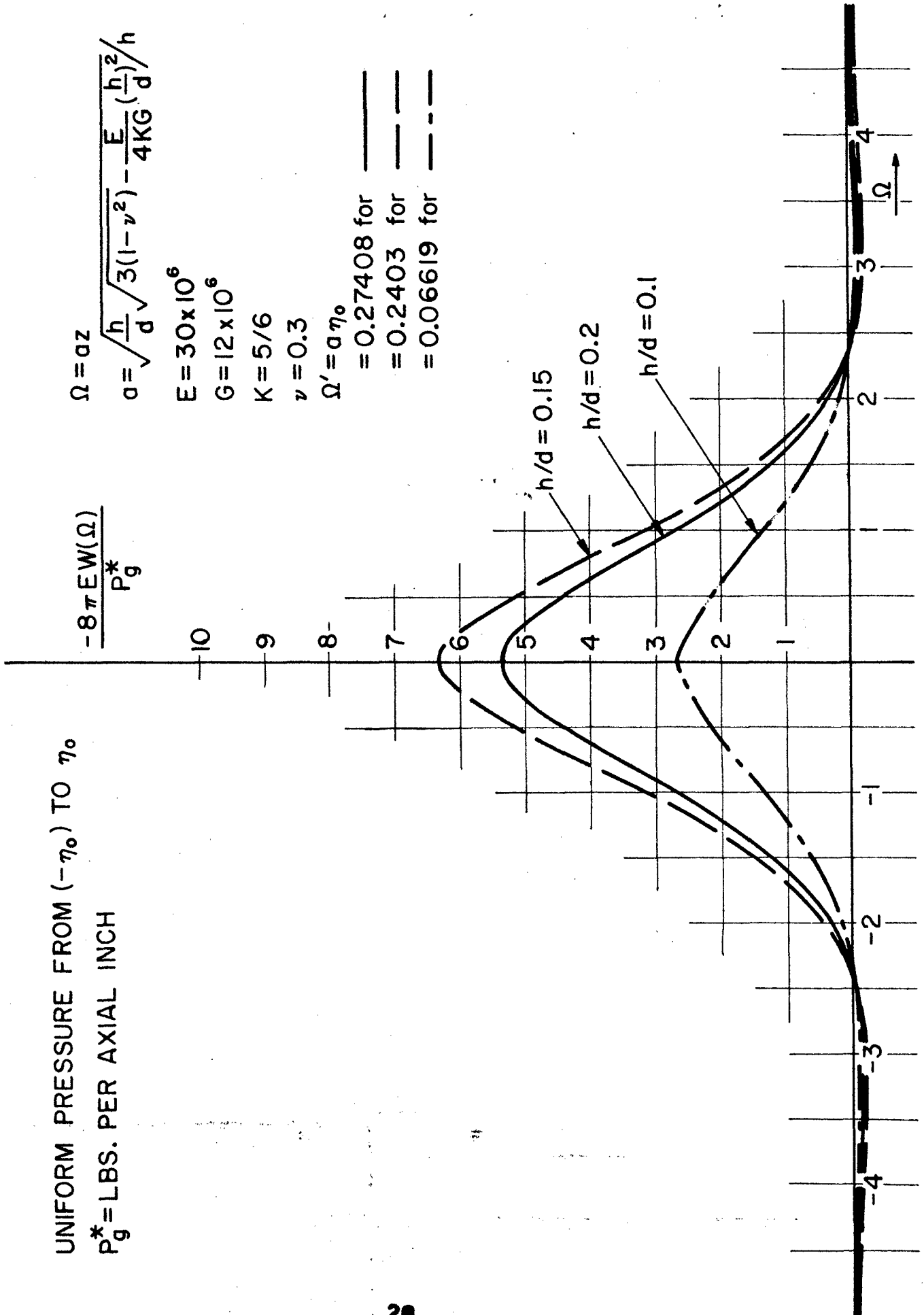
$E = 30 \times 10^6$

$G = 12 \times 10^6$

$K = 5/6$

$\nu = 0.3$

$\Omega' = a\eta_0$

——— $= 0.27408$ for

— — — $= 0.2403$ for

— · — $= 0.06619$ for

h/d = 0.15

h/d = 0.2

h/d = 0.1

$\Omega \longrightarrow$

$$\Omega = a z$$

$$\Omega^1 = a \eta_o$$

Sample tables and curves are shown on pages 23, 24, 25, 26, 27, and 28.

SIMPLIFIED ANALYSIS OF TWO-PIECE SHELL

An analysis was made of a two piece shell described in Picatinny Arsenal drawings AA-38-61 (Base) and AA-38-33 (Body, Rear). This analysis is reported in TN 1-4.

Simplifications were made in the geometry of the shell in order to make it more amenable to analysis. The simplified shell is shown on page 30. The region indicated by P_c represents the threaded section which joins the pieces. P_g and P_b represent gas and band pressures, respectively. The loadings P_g and P_b on the outer section of the shell are transmitted through the threaded section to the inner section.

The general procedure followed in the analysis involved obtaining solutions for the separate shell pieces and then equating deflections at the threaded section to obtain a complete solution. In this manner the value of P_b at which yielding would begin was established.
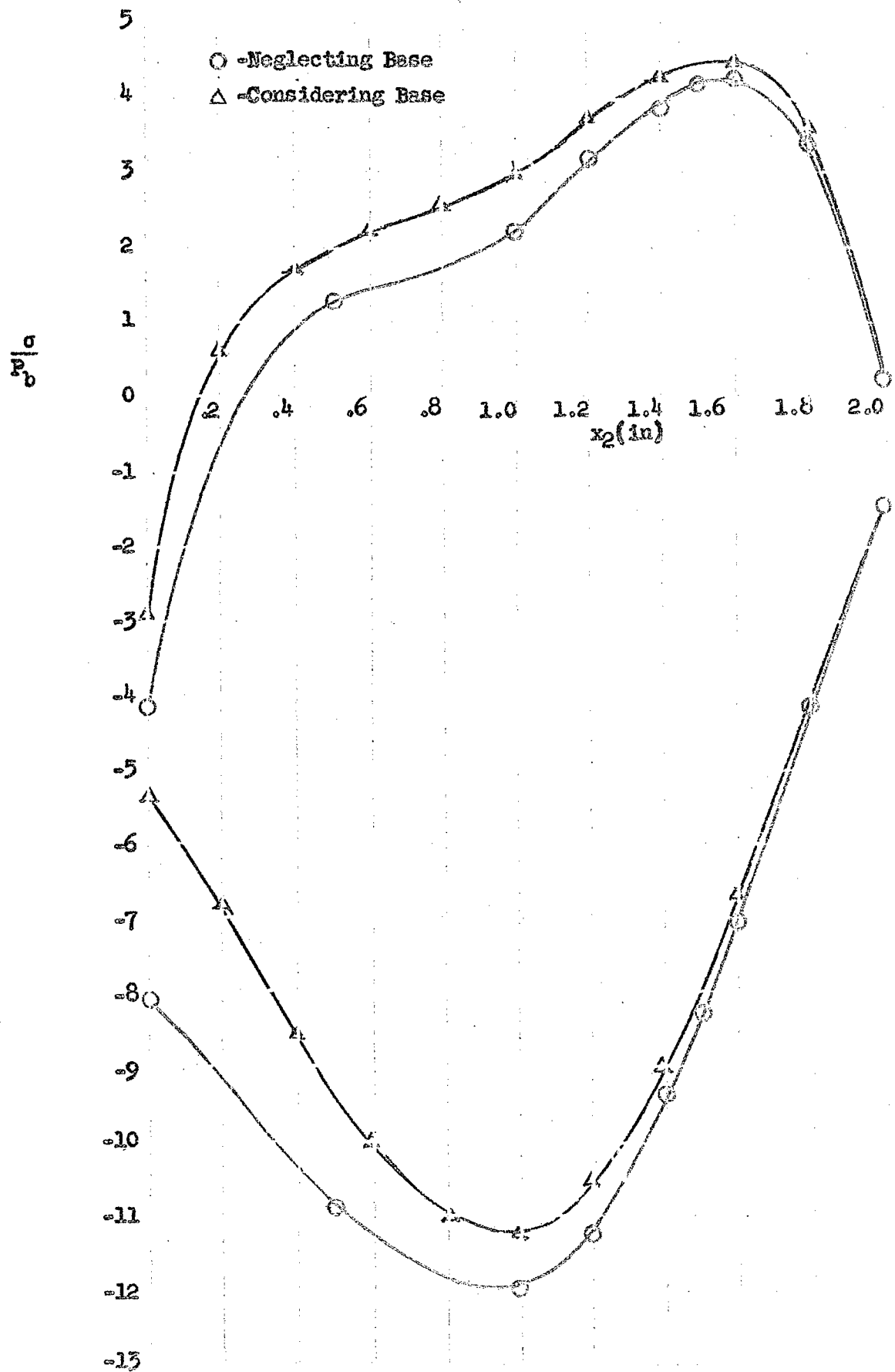
Stresses due to band pressure were solved for two cases: neglecting the base, and including the base. The results are shown
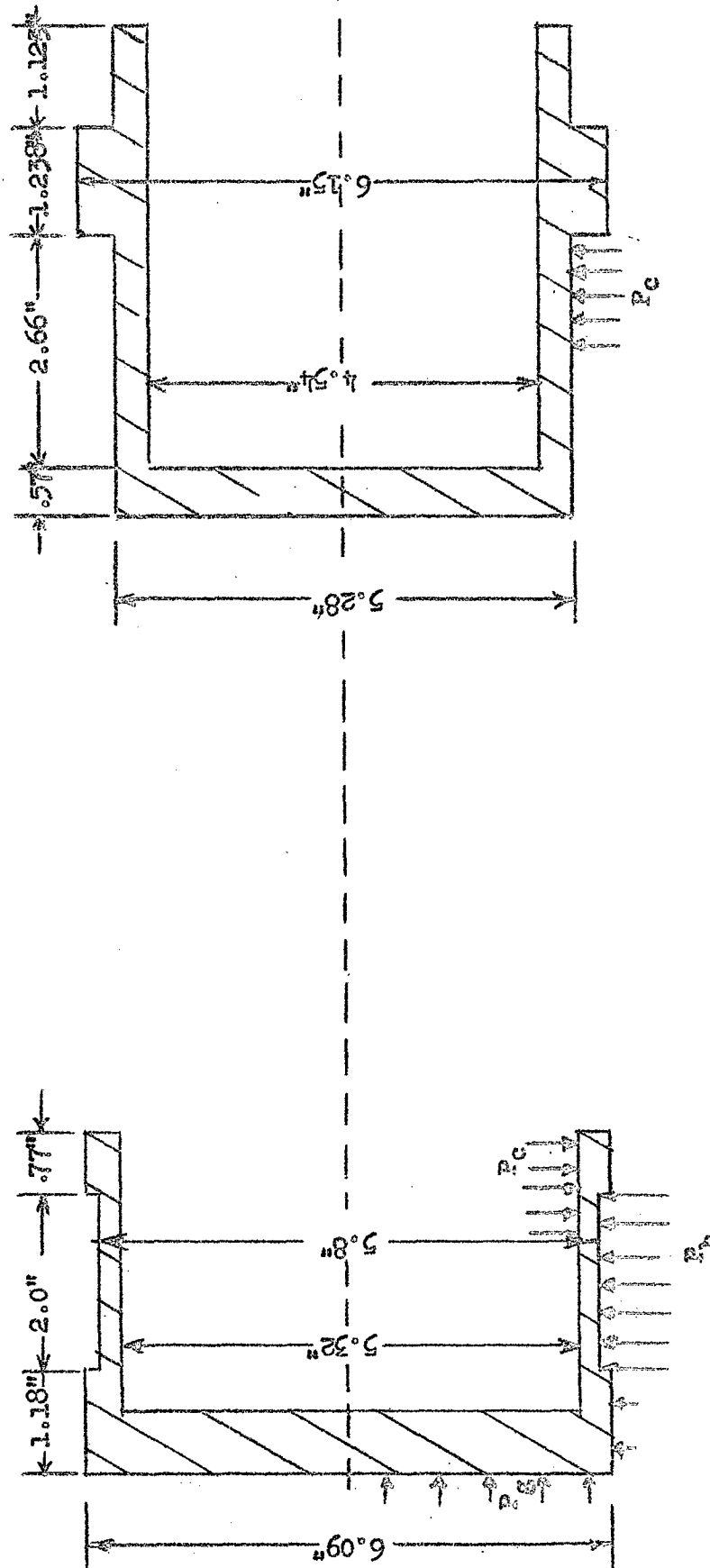
First Modifications of Shell Dimensions - Pressures Shown on Lower Half Only

Ratios of Stress to Band Pressure at Inside of Outer Shell

on page 31. In this set of curves the origin of coordinates
is at the left-hand edge of the band seat.

Stresses in the base due to gas pressure were calculated
for four different conditions of edge support. In addition, the
effect of filler support upon the stresses in the base was
considered and corresponding calculations made.

DISCUSSION

With the completion of the present phase of a Research
and Development Study on Stress-Strain Characteristics of Shells
and High Explosives, considerably more insight has been gained
into the behavior of shell-filler systems.

The general plan of the study has been directed toward
ultimately giving the shell designer analytical and experimental
tools which will enable him to produce rational designs.

The analysis of shells and shell-filler systems is
complicated by several factors, principal among which are: the
inability to accurately obtain rotating band pressures from
theoretical considerations, the yielding of shell walls in the
vicinity of the rotating band, the lack of information concerning
the dynamic behavior of filler materials, and the lack of
information concerning the interaction between shell and filler.

At the present time the most reliable way of obtaining
band pressures is by measuring gun tube strains. Since this
information is always after the fact, i.e., it is not available
before a shell is designed, it is highly desirable that further
effort be expended to develop analytic techniques for deducing
band pressures associated with particular shell design. This
problem involves consideration of the gun-band-shell-filler
system and, while complicated, it appears that a successful
solution can be obtained.

Yielding probably occurs in the vicinity of the
rotating band in most current shell designs and except for

cases where the accompanying deformation causes a malfunction in an internally carried device, such yielding may be considered a part of "good" design. However, at the present time the designer has no idea whatsoever of the amount of yielding present in his design. It seems quite reasonable, on the basis of the present study, that this local yielding could be used to considerable advantage if a rational analysis were available. For this reason work was begun on an analytic treatment of this problem.

The effect of strain rate on filler properties has been formulated theoretically in terms of a visco-elastic solid. Solutions have been obtained for the case where the filler is not constrained along its sides and for the case of a suddenly applied ring of pressure. Experimental techniques have been designed and instruments built for measuring the material properties required in the analysis. It is not certain, on the basis of firing strain measurements made up to the present time, that filler behavior is markedly affected (i.e., material properties markedly different) during firing. However, experimental techniques have now been developed to the point where this question can be answered and continued effort should result in a successful conclusion.

Measurements to determine the interaction between shell and filler have resulted in the conclusion that even shells having quite thin walls very probably receive, in the region of the rotating band, little, if any, support from a filler;

in fact, it is doubtful whether in many designs the shell actually deflects enough to apply any load at all to the filler. On the other hand it seems probable that the base receives considerable support from the filler and may, therefore, be of considerable importance in the design of the base. This effect will receive further study.

One of the important objectives of the present study was to establish whether the Arsenal should contract for the construction of a triaxial tester to be used in connection with filler materials. It is felt that enough information has now been obtained upon which a recommendation can be based, and it is recommended that such a device not be purchased. This recommendation is based on the following considerations: 1) the bulk properties obtained from triaxial tests cannot be incorporated in analysis based upon the theory of elasticity. There is, therefore, no way of incorporating in a general analytic procedure, information obtained in this way. 2) Analytic procedures for the case of a visco-elastic cylinder accelerated from one end and a visco-elastic cylinder subject to a ring load have been obtained. An inexpensive prototype of an instrument for measuring the viscosity of filler materials has been constructed. 3) There is evidence that fillers may be modelled as visco-elastic cylinders accelerated from one end. 4) Experimental techniques have been developed for measuring shell strains during high accelerations; these techniques may be adopted to measuring filler deformation during firing.

It should be mentioned that triaxial tests made for the purpose of investigating the visco-plastic behavior of materials have important application. However, it would not appear that such a function presently would be of great use to the ordance engineer.

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